AIEEE 2007 Mathematics

Q. 1. In a geometric progression consisting of positive terms, each term equals the sum of the next two terms. Then the common ratio of this progression equals

a. $\frac{1}{2}\left(1-\sqrt{5}\right)$
b. $\frac{1}{2}\sqrt{5}$
c. $\sqrt{5}$
d. $\frac{1}{2}\left(\sqrt{5}-1\right)$

Ans. d

Q. 2. If sin $-1\left(\frac{x}{5}\right)+\operatorname{cosec}^{-1}\left(\frac{5}{4}\right)=\frac{\pi}{2}$ then a value of x is.

a. 1
b. 3
c. 4
d. 5

Ans. b

Q. 3. In the binomial expansion of

$(a-b)^n$, $n \geq 5$, *the sum of* 5^{th} *and* 6^{th} *terms in zero, then* $\frac{a}{b}$ *equals*

a. $\frac{5}{n-4}$
b. $\frac{6}{n-5}$
c. $\frac{n-5}{6}$
d. $\frac{n-4}{6}$

Ans. d

Q. 4. The set S : = {1, 2, 3, ……, 12} is to be partitioned into three sets A, B, C of equal size. Thus, $A \cup B \cup C = S, A \cap B = B \cap C = A \cap C = \phi$. The number of ways to partition S is

a. $\frac{12!}{3!(4!)^3}$
b. $\frac{12!}{3!(3!)^4}$
c. $\frac{12!}{(4!)^3}$
d. $\frac{12!}{(3!)^4}$

Ans. **c**

Q. 5. The largest interval lying in $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$ for which the function

$\left[f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)\right]$ *is defined, is*

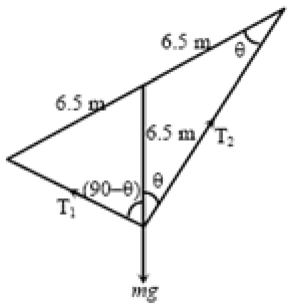
a. $[0, \pi]$
b. $\left(\frac{-\pi}{2}, \frac{\pi}{2}\right)$
c. $\left[-\frac{\pi}{4}, \frac{\pi}{2}\right)$
d. $\left[0, \frac{\pi}{2}\right)$

a.

Ans. **d**

Q. 6. A body weighing 13 kg is suspended by two strings 5 m and 12 m long, their other ends being fastened to the extremities of a rod 13 m long. If the rod be so held that the body hangs immediately below the middle point. The tensions in the strings are

a. 12 kg and 13 kg
b. 5 kg and 5 kg
c. 5kg and 12 kg
d. 5 kg and 13 kg



Ans. c

Q. 7. A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is

a. $\frac{1}{729}$

b. $\frac{8}{9}$

c. $\frac{8}{729}$

d. $\frac{8}{243}$

Ans. d

Q. 8. Consider a family of circles which are passing through the point (- 1, 1) and are tangent to x-axis. If (h, k), are the co-ordinates of the centre of the circles, then the set of values of k is given by the interval

a. $0 < k < \frac{1}{2}$

b. $k \geq \frac{1}{2}$

c. $-\frac{1}{2} \leq k \leq \frac{1}{2}$

d. $\leq \frac{1}{2}$

Ans. 2.

Q. 9. Let L be the line of intersection of the planes 2x + 3y +z = 1 and x + 3y +2z = 2. If L makes an angle α with the positive x-axis, then $\cos \alpha$ equals

a. $\frac{1}{\sqrt{3}}$
b. $\frac{1}{2}$
c. 1
d. $\frac{1}{\sqrt{2}}$

Ans. a

Q. 10. The differential equation of all circles passing through the origin and having their centers on the x-axis is

a. $x^2 = y^2 + xy\frac{dy}{dx}$
b. $x^2 = y^2 + 3xy\frac{dy}{dx}$
c. $y^2 = x^2 + 2xy\frac{dy}{dx}$
d. $y^2 = x^2 - 2xy\frac{dy}{dx}$

Ans. c

Q. 11. If p and q are positive real numbers such that then $p^2 + q^2 = 1,$ the maximum value of (p + q) is

a. 2
b. $1/2$
c. $1/\sqrt{2}$
d. $\sqrt{2}$

Ans. d

Q. 12. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that $AB\,(= a)$ subtends an angle of 60^0 at the foot of the tower, and the angle of elevation of the top of the tower from A or B is 30^0. The height of the tower is

a. $\frac{2a}{\sqrt{3}}$
b. $2a\sqrt{3}$

c. $\frac{a}{\sqrt{3}}$

d. $a\sqrt{3}$

Ans. **c**

Q. 13. The sum of the series $^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots\ldots\_\ldots + {}^{20}C_{10}\ is$

a. $-{}^{20}C_{10}$

b. $\frac{1}{2}\,{}^{20}C_{10}$

c. 0

d. $^{20}C_{10}$

Ans. **b**

Q. 14. The normal to a curve at P(x, y) meets the x-axis at G. If the distance of G from the origin is twice the abscissa of P, then the curve is a

a. ellipse
b. parabola
c. circle
d. hyperbola

Ans. **a, d**

Q. 15. $If\ |z+4| \le 3, then\ the\ \max imum\ value\ of\ |z+1| is$

a. 4
b. 10
c. 6
d. 0

Ans. **c**

Q. 16. The resultant of two forces P N and 3 N is a force of 7 N. If the direction of 3 N force were reversed, the resultant would be $\sqrt{19}\ N$. The value of P is

a. 5 N
b. 6 N
c. 3 N
d. 4 N

Ans. **a**

Q. 17. Two Aeroplanes I and II bomb a target in succession. The probabilities of I and II scoring a hit correctly are 0.3 and 0.2, respectively. The second plane will bomb only if the first misses the target. The probability that the target is hit by the second plane is

a. 0.06
b. 0.14
c. 0.2
d. 0.7

Ans. **None**

Q. 18. $If\ D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix} for\ x \neq 0, y \neq 0\ then\ D\ is$

a. divisible by neither x nor y
b. divisible by both x and y
c. divisible by x but not y
d. divisible by y but not x

Ans. **b**

Q. 19. For the hyperbola $\frac{x^2}{\cos^2 \alpha} - \frac{y^2}{\sin^2 \alpha} = 1,$ which of the following remains constant when α varies?

a. Eccentricity
b. Directrix
c. Abscissae of vertices
d. Abscissae of foci

Ans. **d**

Q. 20. If a line makes an angle of $\frac{\pi}{4}$ with the positive directions of each of x-axis and y-axis, then the angle that the line makes with the positive direction of the z-axis is

a. $\frac{\pi}{6}$
b. $\frac{\pi}{3}$

c. $\frac{\pi}{4}$

d. $\frac{\pi}{2}$

Ans. **d**

Q. 21. A value of C for which the conclusion of Mean Value Theorem holds for the function $f(x) = \log_e x$ on the interval [1, 3] is

a. $2\log_3 e$

b. $\frac{1}{2}\log_e 3$

c. $\log_3 e$

d. $\log_e 3$

Ans. **a**

Q. 22. The function $f(x) = \tan^{-1}(\sin x + \cos x)$ is an increasing function in

a. $\left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

b. $\left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

c. $\left(0, \frac{\pi}{2}\right)$

d. $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

Ans. **b**

Q. 23. $Let\ A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix}.\ If\ |A^2| = 25,\ then |\alpha|\ equals$

a. 5^2

b. 1

c. $\frac{1}{5}$

d. 5

Ans. **c**

Q. 24. $The\ sum\ of\ series\ \frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-.....\ upto\ \inf\ inity\ is$

a. e^{-2}
b. e^{-1}
c. $e^{-\frac{1}{2}}$
d. $e^{+\frac{1}{2}}$

Ans. **b**

Q. 25. If $\hat{u}\ and\ \hat{v}$ are unit vectors and θ is the acute angle between them, then $2\hat{u}\times 3\hat{v}$ is a unit vector for

a. Exactly two values of θ
b. More than two values of θ
c. No value of θ
d. Exactly one value of θ

Ans. **d**

Q. 26. A particle just clears a wall of height *b* at a distance *a* and strikes the ground at a distance c from the point of projection. The angle of projection is

a. $\tan^{-1}\frac{b}{ac}$
b. 45^0
c. $\tan^{-1}\frac{bc}{a(c-a)}$
d. $\tan^{-1}\frac{bc}{a}$

Ans. **c**

Q. 27. The average marks of boys in a class is 52 and that of girls is 42. The average marks of boys and girls combined is 50. The percentage of boys in the class is

a. 40
b. 20
c. 80
d. 60

Ans. **c**

Q. 28. The equation of a tangent to the parabola $y^2 = 8x$ is $y = x + 2$. The point on this line from which the other tangent to the parabola is perpendicular to the given tangent is

a. (- 1, 1)
b. (0, 2)
c. (2, 4)
d. (- 2, 0)

Ans. **d**

Q.29. If (2, 3, 5) is one end of a diameter of the sphere $x^2 + y^2 + z^2 - 6x - 12y - 2z + 20 = 0$, then the coordinates of the other end of the diameter are

a. (4, 9, - 3)
b. (4, - 3, 3)
c. (4, 3, 5)
d. (4, 3, - 3)

Ans. **a**

Q. 30. *Let* $\bar{a} = \hat{i} + \hat{j} + \hat{k}, \bar{b} = \hat{i} - \hat{j} + 2\hat{k}$ *and* $\bar{c} = x\hat{i} + (x-2)\hat{j} - \hat{k}$. *If the vector* $\bar{c}$ *lies in the plane of* $\bar{a}$ *and* $\bar{b}$, *then* x *equals*

a. 0
b. 1
c. – 4
d. – 2

Ans. **d**

Q. 31. Let A (h, k), B (1, 1) and C (2, 1) be the vertices of a right angled triangle with AC as its hypotenuse. If the area of the triangle is 1, then the set of values which 'k' can take is given by

a. $\{1, 3\}$
b. $\{0, 2\}$
c. $\{-1, 3\}$
d. $\{-3, -2\}$

Ans. **c**

Q. 32. $Let\ P = (-1, 0), Q = (0, 0)\ and\ R = (3, 3\sqrt{3})$ be three points. The equation of the bisector of the angle PQR is

a. $\sqrt{3}x + y = 0$
b. $x + \frac{\sqrt{3}}{2}y = 0$
c. $\frac{\sqrt{3}}{2}x + y = 0$
d. $x + \sqrt{3}y = 0$

Ans. a

Q. 33. If one of the lines of $my^2 + (1 - m^2)xy - mx^2 = 0$ is a bisector of the angle between the lines xy = 0, then m is

a. $-\frac{1}{2}$
b. -2
c. 1
d. 2

Ans. c

Q. 34. $Let\ F(x) = f(x) + f\left(\frac{1}{x}\right), where\ f(x) = \int_1^x \frac{\log t}{1+t}dt.\ Then\ F(e)\ equals$

a. $\frac{1}{2}$
b. 0
c. 1
d. 2

Ans. a

Q. 35. $Let\ f : R \to R\ be\ a\ function\ defined\ by\ f(x) = Min\{x+1, |x|+1\}$. Then which of the following is true?

a. $f(x) \geq 1\ for\ all\ x \in R$
b. f (x) is not differentiable at x = 1.
c. f (x) is differentiable everywhere
d. f (x) is not differentiable at x = 0.

Ans. **c**

Q. 36. $The\ function\ f : R \setminus \{0\} \rightarrow R\ given\ by\ f(x) = \frac{1}{x} - \frac{2}{e^{2x} - 1}$ can be made continuous at x = 0 by defining f (0) as

a. 2
b. – 1
c. 0
d. 1

Ans. **d**

Q. 37. $\int \frac{dx}{\cos x + \sqrt{3} \sin x}\ equals$

a. $\frac{1}{2} \log \tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$
b. $\frac{1}{2} \log \tan\left(\frac{x}{2} - \frac{\pi}{12}\right) + C$
c. $\frac{1}{2} \log \tan\left(\frac{x}{2} - \frac{\pi}{12}\right) + C$
d. $\log \tan\left(\frac{x}{2} + \frac{\pi}{12}\right) + C$

Ans. **a**

Q. 38. The area enclosed between the curves $y^2 = x\ and\ y = |x|\ is$

a. $\frac{2}{3}$
b. 1
c. $\frac{1}{6}$
d. $\frac{1}{3}$

Ans. **c**

Q. 39. If the difference between the roots of the equation $x^2 + ax + 1 = 0\ is\ less\ than\ \sqrt{5}$ then the set of possible values of a is

a. $(-3, 3)$
b. $(-3, \infty)$
c. $(3, \infty)$
d. $-\infty, -3$

Ans. **a**

SOLUTIONS

1. Let geometric progression is a, ar, ar2,………… ; (a, r > 0)

$$\because a = ar + ar^2 \Rightarrow r^2 + r - 1 = 0 \Rightarrow r = \frac{-1 \pm \sqrt{5}}{2} \Rightarrow r = \frac{\sqrt{5}-1}{2}$$

Correct choice: (4)

2. $$\sin -1\left(\frac{x}{5}\right) + \operatorname{cosec}^{-1}\left(\frac{5}{4}\right) = \frac{\pi}{2} \Rightarrow \sin^{-1}\left(\frac{x}{5}\right) + \cos^{-1}\left(\frac{3}{5}\right) = \frac{\pi}{2} \Rightarrow x = 3$$

Correct choice: (2)

3. $$\because T_5 + T_6 = 0 \Rightarrow {}^nc_4\, a^{n-4}(-b)^4 + {}^nc_5\, a^{n-5}(-b)^5 = 0 \Rightarrow \frac{a^{n-4}\, b^4}{a^{n-5}\, b^5} = \frac{{}^nC_5}{{}^nC_4} \Rightarrow \frac{a}{b} = \frac{n-4}{5}$$

Correct choice: (4)

4. $\because$ 12 different objects are to be divided into 3 groups of equal size, which are named as set A, B and C. $\Rightarrow \textit{Number of ways} = \frac{12!}{(4!)^3 \cdot 3!} \times 3! = \frac{(12)!}{(4!)^3}$ Correct choice: (3)

5. **Sol:**

$$f(x) = 4^{-x^2} + \cos^{-1}\left(\frac{x}{2} - 1\right) + \log(\cos x)$$

4^{-x^2} *is defined for* $\forall\ x \in R$..........(i)

$\cos^{-1}\left(\frac{x}{2} - 1\right)$ *is defined when* $-1 \le \frac{x}{2} - 1 \le 1$, *i.e when* $0 \le x \le 4$.........(ii)

$\log\ \cos x$ *is defined when* $\cos x > 0$, *i.e when* $x \in \left(-\frac{x}{2}, \frac{x}{2}\right)$...........(iii)

from (i), (ii) *and* (iii), *wehave domain of* $f(x)$ *as* $\left[0, \frac{\pi}{2}\right)$

Correct choice: (4)

6. **Sol:**

$$T_2 \cos\theta + T_1 \sin\theta = mg; \quad T_2 \sin\theta + T_1 \cos\theta T_2 = mg\cos\theta$$

$$T_1 = mg\sin\theta; \tan\theta = \frac{5}{12}$$

$$T_1 = (13\,kg)\frac{5}{13} = 5\,kg; T_2 = (13\,kg)\frac{12}{13} = 12\,kg$$

So tension in the strings are 5 kg and 12 kg. Correct choice: (3)

7. Probability of getting sum of nine in a single throw $= \frac{1}{9}$

Probability of getting sum nine exactly two times out of three draws

$$= {}^3C_2\left(\frac{1}{9}\right)^2\left(\frac{8}{9}\right) = \frac{8}{243}.$$

Correct choice: (4)

8. ⊝ Centre of circle is (h, k) and x-axis is tangent. $\Rightarrow$Radius of the family of circle $= |k|$

Also circle passes through

$$(-1, 1) \Rightarrow (1+h)^2 + (1-k)^2 = k^2 \Rightarrow h^2 + 2h + 2 - 2k = 0 \text{ for real } h, k \geq \frac{1}{2}$$

Correct choice: (2)

9. **Sol:**

$$\text{Vector normal to the plane } 2x+3y+z=1 \text{ is } \vec{a} = 2\hat{i} + 3\hat{j} + \hat{k}$$

$$\text{Vector normal to the plane } x+3y+2z=2 \text{ is } \vec{b} = \hat{i} + 3\hat{j} + 2\hat{k}$$

$$\text{Vector parallel to line of intersection of given planes} = \vec{a} \times \vec{b} = 3\hat{i} - 3\hat{j} + 3\hat{k}$$

$$\text{Angle } \alpha \text{ between } 3\hat{i} - 3\hat{j} + 3\hat{k} \text{ and } \hat{i} \text{ is given by } \cos\alpha = \frac{1}{\sqrt{3}}$$

Correct choice: (1)

10. Equation of family of circles passing through origin and having their centers on x-axis is

$x^2 = y^2 - 2xh = 0 ......(i)$

$Differentiating\ w.r.t\ x\ we\ have\ 2x + 2y\frac{dy}{dx} - 2h = 0 ......(ii)$

$E\lim inating\ 'h'\ from\ (i)\ and\ (ii),\ we\ have\ y^2 = x^2 + 2xy\frac{dy}{dx}$

Correct choice: (3)

11. Equation of family of circles passing through origin and having their centres on x-axis is

$x^2 = y^2 - 2xh = 0 ......(i)$

$Differentiating\ w.r.t\ x\ we\ have\ 2x + 2y\frac{dy}{dx} - 2h = 0 ......(ii)$

$E\lim inating\ 'h'\ from\ (i)\ and\ (ii),\ we\ have\ y^2 = x^2 + 2xy\frac{dy}{dx}$

Correct choice: (3)

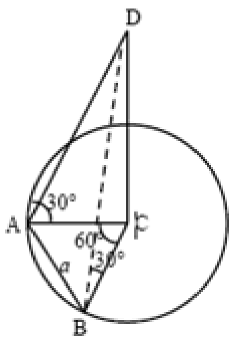
12. **Sol :**

$\Theta \angle ACB = 60^0$

$\Rightarrow Triangle\ ABC\ is\ an\ equilateral\ triangle$

$\Rightarrow Radius\ of\ circle = a$

$Now\ \frac{DC}{AC} = \tan 30^0 \Rightarrow DC = \frac{a}{\sqrt{3}}$



Correct choice: (3)

13.

$$\Theta\ {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - {}^{20}C_3 + \ldots\_\ {}^{20}C_9 + {}^{20}C_{10} - {}^{20}C_{11} + {}^{20}C_{12} - \ldots\ldots + {}^{20}C_{20} = 0$$

$$\Rightarrow 2\left\{{}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - \ldots\ldots - {}^{20}C_9\right\} + {}^{20}C_{10} = 0$$

$$\Rightarrow 2\left\{{}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - \ldots\ldots - {}^{20}C_9 + {}^{20}C_{10}\right\} = {}^{20}C_{10}$$

$$\Rightarrow {}^{20}C_0 - {}^{20}C_1 + {}^{20}C_2 - \ldots\ldots + {}^{20}C_{10} = \frac{1}{2}\ {}^{20}C_{10}$$

Correct choice: (2)

14. **Sol :**

$$\Theta\ |x + my| = 2x \Rightarrow x + my = \pm 2x \Rightarrow ydy = xdx \ or\ ydy$$

$$= -3xdx \Rightarrow \frac{y^2}{2} = \frac{x^2}{2} + (which\ is\ hyperbola)$$

$$or\ \frac{3x^2}{2} + \frac{y^2}{2} = k\ (which\ is\ an\ ellipse)$$

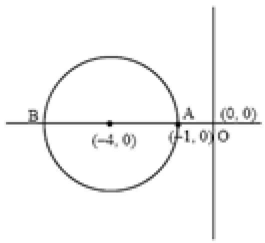
Correct choice: (1, 4)

15.

$$If\ |z + 4| \leq 3, then\ the\ \max imum\ value\ of\ |z + 1|\ is$$

$$\Rightarrow z\ lies\ inside\ or\ on\ the\ circle\ of\ radius\ 3\ and\ centre\ at\ (-4, 0)$$

$$\Rightarrow \max imum\ value\ of\ |z + 1|\ is\ 6.$$



Correct choice: (3)

16. **Sol:**

$$\Theta\ 7^2 = P^2 + 9 + 6P\cos\theta \Rightarrow 6P\cos\theta = 40 - P^2$$

$$and\ 19 = P^2 + 9 + 6P\cos(\pi - \theta) \Rightarrow 19 = P^2 + 9 - 6P\cos\theta$$

$$\Rightarrow 19 = P^2 + 9 - 40 + P^2\ \{u\sin g\ (i)\}$$

$$\Rightarrow P = 5\ N$$

Correct choice: (1)

17. **Sol:**

$P(I) = 0.3.\ P(II) = 0.2$

$\text{Required probability} = P(\bar{I}).\ P(II)\ P(\bar{I})\ P(II) + \ldots\ldots$

$= (0.7)(0.2) + (0.7)(0.8)(0.7)(0.2) + \ldots\ldots$

$= \dfrac{7}{22}$

No choice is correct

18. **Sol:**

$$D = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 1+x & 1 \\ 1 & 1 & 1+y \end{vmatrix} C_3 \to C_3 - C_1$$

$$C_2 \to C_2 - C_1$$

$$= \begin{vmatrix} 1 & 0 & 0 \\ 1 & x & 0 \\ 1 & 0 & y \end{vmatrix}$$

Correct choice: (2)

19. **Sol:**

$\dfrac{x^2}{\cos^2\alpha} - \dfrac{y^2}{\sin^2\alpha} = 1,$

$\because b^2 = a^2(e^2 - 1) \Rightarrow \sin^2\alpha = \cos^2\alpha\,(e^2 - 1) \Rightarrow e^2 = \tan^2\alpha + 1 = \sec^2\alpha$

$\Rightarrow e = \sec\alpha$

$Directrix : x = \pm\dfrac{a}{e} = \pm\cos^2\alpha$

$Abscissae\ of\ vertices = \pm a = \pm\cos\alpha$

$ABCISSAE\ F\ FOCI = \pm ae = \pm\cos\alpha\ .\sec\alpha = \pm 1$

Correct choice: (4)

20. **Sol:**

$\Theta \cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1 \quad \Rightarrow \frac{1}{2} + \frac{1}{2} + \cos^2\gamma = 1 \;\; as\;\alpha = \beta = \frac{\pi}{4}$

$\Rightarrow \cos^2\gamma = 0 \Rightarrow \gamma = \frac{\pi}{2}$

Correct choice: (4)

21. $f`(C) = \frac{f(3) - f(1)}{2} \Rightarrow \frac{1}{C} = \frac{\log_e 3}{2} \Rightarrow C = 2\log_3 e$

Correct choice: (1)

22. **Sol:**

$f(x) = \tan^{-1}(\sin x + \cos x) is\ increa\sin g\ if\ (\sin x + \cos x)\ is\ increa\sin g$

$\Rightarrow \cos x - \sin x > 0 \quad \Rightarrow \quad \cos x > \sin x \quad \Rightarrow \quad x \in \left(-\frac{\pi}{2}, \frac{\pi}{4}\right)$

Correct choice: (2)

23. **Sol:**

$$A = \begin{bmatrix} 5 & 5\alpha & \alpha \\ 0 & \alpha & 5\alpha \\ 0 & 0 & 5 \end{bmatrix} \Rightarrow |A.A| = |A||A| = (25\alpha)^2 = 25 \Rightarrow \alpha^2 = \frac{1}{25} \Rightarrow \alpha = \pm\frac{1}{5}$$

Correct choice: (3)

24. **Sol :**

$e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots\ldots. \quad Put\ x = -1$

$\Rightarrow \frac{1}{2!} - \frac{1}{3!} + \frac{1}{4!} - \ldots\ldots\ldots = e^{-1}$

Correct choice: (2)

25. **Sol :**

$$|2\hat{u}\times 3\hat{v}|\; 6\,|\hat{u}\times\hat{v}| = 1$$

$$\Rightarrow |\hat{u}\times\hat{v}| = \frac{1}{6} \Rightarrow \sin\ \theta = \pm\frac{1}{6}$$

As θ is acute angle for $\sin\ \theta = \frac{1}{6}$. *So θ can take only one Value.*

Correct choice: (4)

26.

$$b = a\ \tan\ \alpha - \frac{1}{2}\frac{ga^2}{u^2\ \cos^2\ \alpha}\ (\textit{equation of trajectory})$$

$$So,\ c = \frac{u^2\ \sin\ 2\alpha}{g} \Rightarrow \frac{g}{u^2} = \frac{\sin\ 2\alpha}{c}$$

$$So,\ b = a\ \tan\ \alpha - \frac{1}{2}\times\frac{a^2\ \sin\ 2\alpha}{c\ \cos^2\ \alpha} \Rightarrow b = a\ \tan\ \alpha - \frac{a^2}{c}\tan\ \alpha$$

$$\Rightarrow b = \left(\frac{ac - a^2}{c}\right)\tan\ \alpha \Rightarrow \tan\ \alpha = \frac{bc}{a\,(c-a)}$$

Correct choice: (3)

27. Let there are x boys and y girls in the class. So, total marks are 52 x + 42 y and according to second condition total marks = (x + y) 50
Now, $52\ x + 42\ y = 50\ x + 50\ y \Rightarrow 2x = 8y \Rightarrow x = 4$
So percentage of boys is 80.

Correct choice: (3)

28. Point of intersection of two perpendicular tangents to the parabola lies on directrix of the parabola. Equation of direct rix is x + 2 = 0
So point is (- 2, 0) Correct choice: (4)

29. Centre (3, 6, 1)

Let other end is (α, β, γ)

$$So,\ \frac{\alpha+2}{2} = 3,\ \frac{\beta+3}{2} = 6,\ \frac{\gamma+5}{2} = 1 \Rightarrow \alpha = 4,\ \beta = 9,\ \gamma = -3$$

Correct choice: (1)

30.

$$\begin{vmatrix} 1 & 1 & 1 \\ 1 & -1 & 2 \\ x & x-2 & -1 \end{vmatrix} = 0 \Rightarrow 1\left(1-2\left(x-2\right)\right)-1\left(-1-2x\right)+1\left(x-2+x\right)=0$$

$$\Rightarrow 2x+4=0 \Rightarrow x=-2$$

Correct choice: (4)

31. $A=\frac{1}{2}.1.\left|k-1\right|=1 \Rightarrow k-1=2 \text{ or } -2 \Rightarrow k=3,-1$

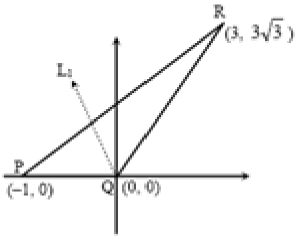
Correct choice: (3)

32.

$Slope\ of\ QR=\sqrt{3}\ .\ So.\ \angle PQR=120^0$

$So\ m_{L1}=\tan 120^0=-\sqrt{3}$

$So\ equation\ of\ L_1\ is\ y=-\sqrt{3}x$



Correct choice: (1)

33. $my^2+\left(1-m^2\right)xy-mx^2=0 \Rightarrow \left(y-mx\right)\left(my+x\right)0 \Rightarrow y=mx,-\frac{1}{m}x$

$So\ m=1\ or\ -1$

Correct choice: (3)

34. $F\left(x\right)=\int_1^x \frac{\log t}{1+t}dt+\int_1^{\frac{1}{x}} \frac{\log t}{1+t}dt=\int_1^x \frac{\log t}{1+t}dt+\int_1^x \frac{\log t}{t.\left(1+t\right)}dt$

$=\int_1^x \frac{\log t}{1+t}dt\ \frac{\left(\log_e x\right)^2}{2}\ .\ So\ F\left(e\right)=\frac{1}{2}$

Correct choice: (1)

35. $f(x) = x + 1, \forall\ x \in R$

Correct choice: (3)

36.

$$f(0) = \lim_{x \to 0} \frac{e^{2x} - 1\ 2x}{x\left(e^{2x} - 1\right)} = 2 \lim_{1 \to 0} \frac{e^{t} - 1 -}{t\,.\left(e^{t} - 1\right)}$$

$$= 2 \lim_{1 \to 0} \frac{e^{t} - 1}{te^{t} + e^{t} - 1} \qquad \textit{By L'Hospital Rule}$$

$$= 2 \lim_{1 \to 0} \frac{e^{t} - 1}{te^{t} + e^{t} + e^{t}} \qquad \textit{Again by L'Hospital Rule}$$

$$= 2 \times \frac{1}{2} = 1$$

Correct choice: (4)

37.

$$\int \frac{dx}{\cos x + \sqrt{3} \sin x} \frac{1}{2} \int \frac{dx}{\frac{\sqrt{3}}{2} \sin x + \frac{1}{2} \cos x} = \frac{1}{2} \int \frac{dx}{\sin\left(\frac{\pi}{6} + x\right)}$$

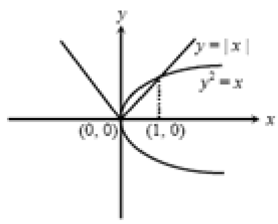
$$\Rightarrow \frac{1}{4} \int \frac{dx}{\sin\left(\frac{\pi}{12} + \frac{x}{2}\right) \cos\left(\frac{\pi}{12} + \frac{x}{2}\right)} = \frac{1}{2} \log \tan\left(\frac{\pi}{12} + \frac{x}{2}\right) + C$$

Correct choice: (1)

38.

$y^2 = x \textit{ and } y = |x| \Rightarrow x^2 = x \Rightarrow x = 0 \textit{ or } 1$

$$\text{Required area} = \int_0^1 \left(\sqrt{x} - x\right) dx = \left[\frac{2}{3} x^{3/2} - \frac{x^2}{2}\right]_0^1 = \frac{2}{3} - \frac{1}{2} = \frac{1}{6}$$



Correct choice: (3)

39. $\alpha + \beta = -a$

$|\alpha - \beta| < \sqrt{5} \Rightarrow (\alpha - \beta)^2 < 5 \Rightarrow a^2 - 4 < 5 \Rightarrow a \in (-3, 3)$

Correct choice: (1)